OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE UNIT **4751/01**

MATHEMATICS (MEI)

Introduction to Advanced Mathematics (C1)

THURSDAY 7JUNE 2007

Morning
Time: 1 hour 30 minutes

Additional materials:
Answer booklet (8 pages)
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are **not** permitted to use a calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

WARNING

You are not allowed to use a calculator in this paper

This document consists of **4** printed pages.

[Turn over

Section A (36 marks)

1 Solve the inequality $1 - 2x < 4 + 3x$. [3]

2 Make t the subject of the formula $s = \frac{1}{2}at^2$. [3]

3 The converse of the statement 'P $\Rightarrow$ Q' is 'Q $\Rightarrow$ P'.

Write down the converse of the following statement.

'n is an odd integer $\Rightarrow$ $2n$ is an even integer.'

Show that this converse is false. [2]

4 You are given that $\mathrm{f}(x) = x^3 + kx + c$. The value of $\mathrm{f}(0)$ is 6, and $x - 2$ is a factor of $\mathrm{f}(x)$.

Find the values of k and c. [3]

5 **(i)** Find a, given that $a^3 = 64x^{12}y^3$. [2]

(ii) Find the value of $\left(\frac{1}{2}\right)^{-5}$. [2]

6 Find the coefficient of x^3 in the expansion of $(3 - 2x)^5$. [4]

7 Solve the equation $\dfrac{4x+5}{2x} = -3$. [3]

8 **(i)** Simplify $\sqrt{98} - \sqrt{50}$. [2]

(ii) Express $\dfrac{6\sqrt{5}}{2+\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers. [3]

9 **(i)** A curve has equation $y = x^2 - 4$. Find the x-coordinates of the points on the curve where $y = 21$. [2]

(ii) The curve $y = x^2 - 4$ is translated by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

Write down an equation for the translated curve. You need not simplify your answer. [2]

10 The triangle shown in Fig. 10 has height $(x + 1)$ cm and base $(2x - 3)$ cm. Its area is $9\,\text{cm}^2$.

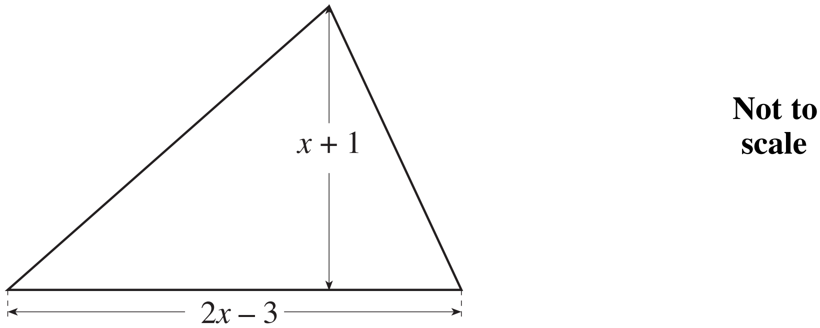

Fig. 10

(i) Show that $2x^2 - x - 21 = 0$. [2]

(ii) By factorising, solve the equation $2x^2 - x - 21 = 0$. Hence find the height and base of the triangle. [3]

Section B (36 marks)

11

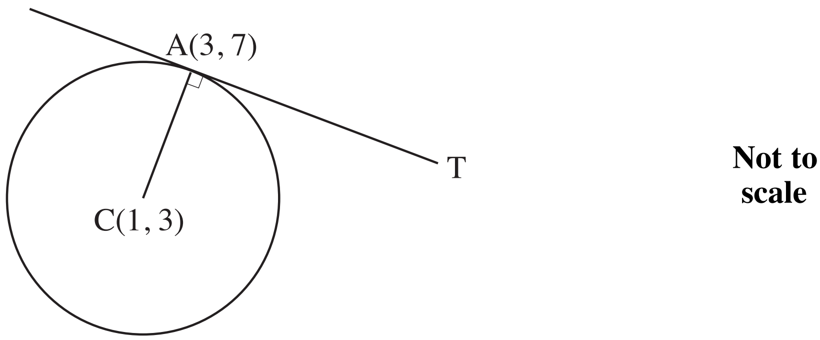

Fig. 11

A circle has centre $C(1, 3)$ and passes through the point $A(3, 7)$ as shown in Fig. 11.

(i) Show that the equation of the tangent at A is $x + 2y = 17$. [4]

(ii) The line with equation $y = 2x - 9$ intersects this tangent at the point T.

Find the coordinates of T. [3]

(iii) The equation of the circle is $(x - 1)^2 + (y - 3)^2 = 20$.

Show that the line with equation $y = 2x - 9$ is a tangent to the circle. Give the coordinates of the point where this tangent touches the circle. [5]

12 **(i)** Write $4x^2 - 24x + 27$ in the form $a(x - b)^2 + c$. [4]

(ii) State the coordinates of the minimum point on the curve $y = 4x^2 - 24x + 27$. [2]

(iii) Solve the equation $4x^2 - 24x + 27 = 0$. [3]

(iv) Sketch the graph of the curve $y = 4x^2 - 24x + 27$. [3]

13 A cubic polynomial is given by $f(x) = 2x^3 - x^2 - 11x - 12$.

(i) Show that $(x - 3)(2x^2 + 5x + 4) = 2x^3 - x^2 - 11x - 12$.

Hence show that $f(x) = 0$ has exactly one real root. [4]

(ii) Show that $x = 2$ is a root of the equation $f(x) = -22$ and find the other roots of this equation. [5]

(iii) Using the results from the previous parts, sketch the graph of $y = f(x)$. [3]

Mark Scheme 4751
June 2007

Section A

1	$x > -0.6$ o.e. eg $-3/5 < x$ isw	3	M2 for $-3 < 5x$ or $x > \frac{3}{-5}$ or M1 for $-5x < 3$ or $k < 5x$ or $-3 < kx$ [condone $\le$ for Ms]; if 0, allow SC1 for -0.6 found	3
2	$t = [\pm]\sqrt{\frac{2s}{a}}$ o.e.	3	B2 for t omitted or $t = \sqrt{\frac{s}{\frac{1}{2}a}}$ o.e. M1 for correct constructive first step in rearrangement and M1 (indep) for finding sq rt of their t^2	3
3	'If $2n$ is an even integer, then n is an odd integer' showing wrong eg 'if n is an even integer, $2n$ is an even integer'	1 1	or: $2n$ an even integer $\Rightarrow$ n an odd integer or counterexample eg $n = 2$ and $2n = 4$ seen [in either order]	2
4	$c = 6$ $k = -7$	1 2	M1 for f(2) = 0 used or for long division as far as $x^3 - 2x^2$ in working	3
5	(i) $4x^4y$ (ii) 32	2 2	M1 for two elements correct; condone y^1 M1 for $\left(\frac{2}{1}\right)^5$ or 2^5 soi or $\left(\frac{1}{32}\right)^{-1}$ or $\frac{1}{\frac{1}{32}}$	4
6	-720 [x^3]	4	B3 for 720; M1 for each of 3^2 and $\pm 2^3$ or $(-2x)^3$ or $(2x)^3$, and M1 for 10 or (5×4×3)/(3×2×1) or for 1 5 10 10 5 1 seen but not for $^5C_3$	4
7	$\frac{-5}{10}$ o.e. isw	3	M1 for $4x + 5 = 2x \times -3$ and M1 for $10x = -5$ o.e. <u>or</u> M1 for $2 + \frac{5}{2x} = -3$ and M1 for $\frac{5}{2x} = -5$ o.e.	3
8	(i) $2\sqrt{2}$ or $\sqrt{8}$ (ii) $30 - 12\sqrt{5}$	2 3	M1 for $7\sqrt{2}$ or $5\sqrt{2}$ seen M1 for attempt to multiply num. and denom. by $2 - \sqrt{5}$ and M1 (dep) for denom -1 or $4 - 5$ soi or for numerator $12\sqrt{5} - 30$	5
9	(i) ± 5 (ii) $y = (x - 2)^2 - 4$ or $y = x^2 - 4x$ o.e. isw	2 2	B1 for one soln M1 if y omitted or for $y = (x + 2)^2 - 4$ or $y = x^2 + 4x$ o.e.	4
10	(i) ½×$(x + 1)(2x - 3) = 9$ o.e. $2x^2 - x - 3 = 18$ or $x^2 - ½x - 3/2 = 9$ (ii) $(2x - 7)(x + 3)$ -3 and 7/2 o.e. or ft their factors base 4, height 4.5 o.e. cao	M1 A1 B1 B1 B1	for clear algebraic use of ½ bh; condone $(x + 1)(2x - 3) = 18$ allow x terms uncollected. NB ans $2x^2 - x - 21 = 0$ given NB B0 for formula or comp. sq. if factors seen, allow omission of -3 B0 if also give $b = -9$, $h = -2$	5

Section B

11	**i**	grad AC = $\frac{7-3}{3-1}$ or 4/2 o.e.[= 2]	M1	not from using − ½	
		so grad AT = – ½	M1	or ft their grad AC [for use of $m_1m_2 = -1$]	
		eqn of AT is $y - 7 = -\frac{1}{2}(x - 3)$	M1	or subst (3, 7) in $y = -\frac{1}{2}x + c$ or in $2y + x = 17$; allow ft from their grad of AT, except 2 (may be AC not AT)	
		one correct constructive step towards $x + 2y = 17$ [ans given]	M1	or working back from given line to $y = -\frac{1}{2}x + 8.5$ o.e.	4
	ii	$x + 2(2x - 9) = 17$	M1	attempt at subst for x or y or elimination	
		$5x - 18 = 17$ or $5x = 35$ o.e.	A1	allow $2.5x = 17.5$ etc	
		$x = 7$ and $y = 5$ [so (7, 5)]	B1	graphically: allow M2 for both lines correct or showing (7, 5) fits both lines	3
	iii	$(x - 1)^2 + (2x - 12)^2 = 20$	M1	subst $2x - 9$ for y [oe for x]	
		$5x^2 - 50x + 125[= 0]$	M1	rearranging to 0; condone one error	
		$(x - 5)^2 = 0$	A1	showing 5 is root and only root	
		equal roots so tangent	B1	explicit statement of condition needed (may be obtained earlier in part) or showing line is perp. to radius at point of contact	
		(5, 1)	B1	condone $x = 5$, $y = 1$	
		<u>or</u>			
		$y - 3 = -\frac{1}{2}(x - 1)$ o.e. seen	M1	or if $y = 2x - 9$ is tgt then line through C with gradient −½ is radius	
		subst or elim. with $y = 2x - 9$	M1		
		$x = 5$	A1		
		(5,1)	B1		
		showing (5, 1) on circle	B1	or showing distance between (1, 3) and (5, 1) = $\sqrt{20}$	5

12	**i**	$4(x-3)^2 - 9$	4	1 for $a = 4$, 1 for $b = 3$, 2 for $c = -9$ or M1 for $27 - 4 \times 3^2$ or $\frac{27}{4} - 3^2 [= -\frac{9}{4}]$	4
	ii	min at (3, –9) or ft from (i)	B2	1 for each coord [e.g. may start again and use calculus to obtain $x = 3$]	2
	iii	$(2x-3)(2x-9)$	M1	attempt at factorising or formula or use of their (i) to sq rt stage	
		$x = 1.5$ or 4.5 o.e.	A2	A1 for 1 correct; accept fractional equivs eg 36/8 and 12/8	3
	iv	sketch of quadratic the right way up	M1		
		crosses x axis at 1.5 and 4.5 or ft crosses y axis at 27	A1 B1	allow unsimplified shown on graph or in table etc; condone not extending to negative x	3
13	**i**	$2x^3 + 5x^2 + 4x - 6x^2 - 15x - 12$	1	for correct interim step; allow correct long division of f(x) by (x − 3) to obtain $2x^2 + 5x + 4$ with no remainder	
		3 is root use of $b^2 - 4ac$ $5^2 - 4 \times 2 \times 4$ or −7 and [negative] implies no real root	B1 M1 A1	allow f(3) = 0 shown or equivalents for M1 and A1 using formula or completing square	4
	ii	divn of f(x) + 22 by x − 2 as far as $2x^3 - 4x^2$ used	M1	or inspection eg $(x-2)(2x^2 \ldots\ldots -5)$	
		$2x^2 + 3x - 5$ obtained $(2x+5)(x-1)$ 1 and −2.5 o.e.	A1 M1 A1 +A1	attempt at factorising/quad. formula/ compl. sq.	
		or			
		$2 \times 2^3 - 2^2 - 11 \times 2 - 12$ $16 - 4 - 22 - 12$ $x = 1$ is a root obtained by factor thm $x = -2.5$ obtained as root	M1 A1 B1 B2	or equivs using f(x) + 22 not just stated	5
	iii	cubic right way up crossing x axis only once (3, 0) and (0, −12) shown	G1 G1 G1	must have turning points must have max and min below x axis at intns with axes or in working (indep of cubic shape); ignore other intns	3

4751: Introduction to Advanced Mathematics (C1)

General Comments

A full spread of marks was seen, with candidates usually attempting all parts of all questions, although occasionally question 13 petered out, perhaps because of time spent earlier on long methods.

The examiners were concerned, as last year, about the long tail of very weak candidates entered for this examination. In contrast, there were excellent scripts seen from those who had learnt the algebraic techniques in the specification and were able to apply them with confidence.

Comments on Individual Questions

Section A

1 Many gained full marks here, although some weak candidates did not have a good strategy for solving inequalities. Those who collected terms to obtain $-3 < 5x$ were usually more successful than those who chose $-5x < 3$ and then often failed to reverse the inequality when dividing by a negative number.

2 There were many correct answers to this rearrangement, although some were careless in their positioning of the square root sign, not making it clear that it included the numerator as well as denominator. Candidates who obtained $\sqrt{\dfrac{s}{\frac{1}{2}a}}$ were expected to simplify this. Weak candidates often made errors in the first step of the rearrangement – for instance those who found the square root as their first step rarely did so correctly.

3 This question exposed a limited understanding of $\Rightarrow$ and $\Leftarrow$, although the wording of the question helped many to write the converse. Most candidates gained the second mark though citing a counter-example rather than using the argument that an even number multiplied by 2 is also even.

4 Some did not appreciate the significance of F(0) = 6 and so were unable to find c. A common error was to get as far as f(2) = 8 + 2k + 6 but then not equate this to 0, often using 6 instead. As usual in this type of question, some evaluated f(−2) instead of f(2).

5 The first part of this question on indices was often well done. The main errors were $4x^9y$ instead of $4x^4y$ or problems in finding $\sqrt[3]{64}$. In part (ii), many candidates did not cope well with the negative index. Few used $\left(\dfrac{2}{1}\right)^5$ and a common wrong answer was $\dfrac{1}{32}$ instead of 32. Some weaker candidates used decimals and spent a while calculating 0.03125, for which they received no credit.

6 A few candidates did not attempt to solve this expansion, and some spent unnecessary time in calculating all the terms rather than just the requested term in x^3. As expected with this topic, the most common error was to forget to cube the coefficient of x, although coping correctly with the other constituent parts, so that an answer of (−)180 instead of −720 was common.

7 This question produced a good spread of marks. Those who wrongly 'cancelled' $4x$ and $2x$ at the start did not gain any credit. After a correct first couple of steps, many made errors in proceeding from $10x + 5 = 0$, with 0.5 and ±2 being common wrong answers.

8 In the first part, most candidates realised that they had to break down the 98 and 50 into factors, but some became muddled with their roots, giving $2\sqrt{5}$ instead of $5\sqrt{2}$, for example. A common wrong answer from weaker candidates was $\sqrt{48}$.

In the second part, many realised the need to multiply both numerator and denominator by $2 - \sqrt{5}$, but errors in doing so were frequent.

9 The first part was often correct, but many candidates ignored the hint and gave only one root of $x^2 = 25$. The second part was poorly done, with most candidates not giving the correct equation of the translated curve. Common errors were $y = x^2 - 2$, $y = x^2 - 6$, $y = (x^2 - 2) - 4$.

10 A surprising number of candidates did not know where to start in part (i), in spite of the diagram given. Those who did correctly start with the area of a triangle being ½ base × height were usually able to proceed correctly to gain their two marks.

In part (ii), some candidates used the quadratic formula, in spite of the instruction to solve by factorising. However, most managed to factorise correctly and realised that only the positive value of x yielded a practical result.

Section B

11 (i) Most candidates worked confidently towards the given answer and gained full marks. Some of the weaker candidates just showed that (3, 7) was on the given line; others tried a bit of working backwards and were sometimes then successful in appreciating they needed to show independently that the gradient of AC was 2 and the gradient of the tangent therefore $-\frac{1}{2}$. This part was one of the best done questions in section B.

(ii) Those who chose to eliminate rather than substitute for *y* often made errors, particularly those who found both equations in terms of *y* and did not cope with the resulting fractions. However, many gained full marks here.

(iii) Many candidates knew what to do, but errors in substitution into the equation of the circle and subsequent simplification were frequent. As a result, many candidates gained only the method marks in this part. Of those who successfully found the intersection as (5, 1), some did not mention that the equal roots showed that the line was a tangent.

12 This was probably the best-attempted section B question as a whole, with the more able candidates finding it straightforward and weaker candidates doing various of the parts independently of each other.

(i) This part caused most problems, with many not able to cope with the coefficient of x^2 not being 1. Common wrong answers $4(x - 3)^2 - 9/4$, $4(x - 12)^2 - 117$ and $4(x - 3)^2 + 18$. In contrast were those who gave the correct answer with little or no working.

(ii) Most candidates used part (i) and were allowed full marks follow through. Some started again and were able to gain full credit by using calculus or the line of symmetry of the graph.

(iii) Those who followed part (i) had problems if they had $4(x - 3)^2 + 18$, but others were able to proceed, although some did not cope with the 4 when taking the square root. The large number of candidates who used the quadratic formula had some hefty arithmetic to do without a calculator and many made errors in doing so. Most successful were those who used factorisation correctly.

(iv) Many candidates now had conflicting information for their graph, but most ignored any that did not appear to fit a parabola-shaped curve. Some candidates started again and constructed a table of values. Where there were two roots given in part (iii), a follow-through was allowed for the intersections with the x-axis. Those who used graph paper often omitted the *y* intercept due to their scale.

13 (i) Most candidates made a correct attempt to multiply out; some showed the given result successfully by long division. However, not many candidates fully showed $x = 3$ to be the only real root. Some showed that f(3) = 0, and others that the discriminant of the quadratic factor was negative, but few did both. A number of candidates said that the root was $(x - 3)$.

(ii) Most candidates started this part by substituting to show that f(2) = −22 or that f(2) + 22 = 0. For weaker candidates, this was often as far as they got. Better candidates often went back to long division of f(x) + 22 and successfully obtained the quadratic factor and hence the roots, using factorisation or the formula.

(iii) Some good graphs of y = f(x) were seen which gained all 3 marks, but frequently, candidates used the results in part (ii) as if they were roots of y = f(x) not y = f(x) + 22, drawing a graph that gained only the first mark. Despite this, a few of these candidates realised that the graph should cross the axes at (3, 0) and (0, −12) and gained another mark for showing this.